

Theoretical Analysis of Modern Portfolio Theory

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Abstract. The Modern portfolio theory has contributed to establishing the fundamental principles of portfolio management, and it is widely used in the finance industry to build diversified investment portfolios. The purpose of the proposed research is to evaluate the effectiveness of modern portfolio theory in the real estate industry by examining empirical evidence and case studies, assisting with real estate developers and property managers to make informed decisions about which properties to invest in and how to manage their real estate portfolios over time. Prior studies have primarily relied on historical data and have been unable to untangle the survey of particular cases and areas owing to recent changes, which may lead to discrepancies in the data and erroneous assessment of investment portfolios. Considering this limitation, the research essay delves explicitly into the effectiveness of modern portfolio theory in the real estate field and how it can assist in developing real estate. Consequently, according to this paper findings and the mathematical derivation, applying modern portfolio theory to real estate is still generally beneficial.

Keywords: Portfolio Theory; Real Estate; Limitation.

1. Introduction

In 1952, American economist Harry Markowitz published Modern Portfolio Theory (MPT). Through extensive research and observation, Markowitz concluded that any investor would choose the security with lower risk out of the two that offered the same rate of return. This further demonstrates that investors must take significant risks to obtain great profits. Similarly, investors frequently retain diverse portfolios due to a fear of taking risks. Markowitz developed an optimization approach for investment portfolios after rigorously examining the features of investment portfolios and quantifying returns and risks. Markowitz believes a portfolio is defined by its constituent securities and their respective weights. The expected return discussed in the MPT is therefore the weighted average of the expected rates of return of each securities. Quantifying the risk associated with a portfolio is critical in addition to assessing out the expected rate of returns. An indicator of a portfolio's risk is the rate of return's standard deviation. These people can understand that the covariance is equivalent to the correlation coefficient between any two stocks [1, 2]. Furthermore, the components of variance, weight, and covariance between securities consist the variance of the portfolio. The covariance decreases with decreasing correlation coefficient, which lowers overall portfolio risk. As a result, the aim of portfolio development should be to choose uncorrelated securities. Also, it is clear from the mathematical growth of the portfolio's variance that adding securities can lower the portfolio's risk [3].

2. Methodology

2.1 The Relationships between Risk and Returns

The MPT hypothesis is that all investors are risk averse, that if two investments offer identical expected returns, investors would prefer the one with less risk. Thus, the theory concludes that investors will accept an increased risk only compensated with increased returns [4].

2.2 Expected Returns

An investor's expected return is the total assets and money they gain or lose from their investment projects. Theoretically, the expected returns play a dominant role in determining investors' investing

behaviors and decisions. However, most of them are based on historical statistics and thus lack the capability to indicate investments' future performance.

2.3 Calculating expected returns

In MPT, the single expected return is generated by multiplying the weight of the asset by its expected returns, and the total expected returns are created by adding together all of the single expected returns for each investment [5,6]. The formula is followed:

$$\mathbb{E}(\mathcal{R}_p) = \sum_i w_i \mathbb{E}(\mathcal{R}_i) \quad (1)$$

Where \mathcal{R}_p is the returns on the portfolio, \mathcal{R}_i is the returns on the asset i , (example of the “single expected return”), and w_i is the weighting of asset i .

The following formula for the Portfolio Risk is shown below,

$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij} = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_i \sigma_j \rho_{ij} \quad (2)$$

Where ρ_{ij} is the periodic returns on the two assets' sample covariance?

2.4 Diversification

A portfolio's risk may be reduced by investing in combinations of assets that are not entirely positively linked. By holding a diverse portfolio of assets, investors may decrease their exposure to the risk associated with specific assets. Diversification may be possible with less risk and the same projected return on the portfolio. Several economists and mathematicians have subsequently confirmed and enhanced Markowitz's mean-variance framework for building optimum investment portfolios by considering the framework's shortcomings.

If all asset pairings have correlations of zero, which indicates that they are fully uncorrelated, the return variance of the portfolio is equal to the sum across all assets of the square of the percentage held in each asset multiplied by the asset's return variance (and the portfolio standard deviation is the square root of this sum).

If all asset pairings have correlations of 1, which indicates that they are entirely positively linked, then the standard deviation of the portfolio return is equal to the sum of the standard deviations of the asset returns, weighted by the proportions held in the portfolio. Given specified portfolio weights and asset return standard deviations, the situation where all correlations are one gives the maximum practicable standard deviation of portfolio return.

3. Real Estate Portfolio Analysis Based on Modern Portfolio Theory

1991 saw the introduction of portfolio theory to real estate investing by Nigel Dubben and Sarah Sayce. They thoroughly covered real estate investment risk, income, and capital portfolio management, highlighting the potential use of portfolio theory in the real estate industry.

Real estate investment portfolio strategies is established on modern portfolio theory should consider project risk in addition to quantitative analytical techniques. Examining the asset portfolio's efficient frontier is an essential analytical technique in the Markowitz model. According to current asset portfolio investing theory, the rational asset selection process maximizes the investor's expected utility. To maximize the projected utility of the investor under certain circumstances, the asset portfolio must be chosen if the market for real estate is split into markets for various assets.

Developers may create various asset portfolios using all or some of their assets in this market. Each asset portfolio can pass the return since each has a set of predicted return rates and variances that correspond to it. Suppose the investment portfolio for the development project could be more effective. Therefore, by adjusting the investment ratio, it may be altered to one that is on the efficient frontier in order to raise anticipated rate of return without raising risk, lower risk without raising expected return, or construct a portfolio that both raises expected return and lowers risk. The goals of the investment portfolio for real estate development projects can thus be summarized as follows: under the level of expected return, the real estate developer can bear the least amount of investment risk through the optimization of the real estate investment portfolio; under a certain risk level, the real estate developer can bear the least amount of investment risk through the optimization of the real estate investment portfolio to make it possible for real estate developers to get the maximum anticipated advantages [7].

4. Construction of Real Estate Portfolio Model

There are two different types of personal investment risks in real estate. One is the market risk, sometimes referred to as systemic risk, which is tied to the entire market and is triggered by certain political and economic developments that have an influence on all real estate at once. Investment activities make up one type of risks; the other is a non-systematic risk that solely pertains to specific real estate investments and is unrelated to the market. Only systematic risk, according to the standard CAPM, adds to the overall risk of an investment portfolio and generates the anticipated return proportionate to its involvement. The contribution of the systematic risk may be expressed as a coefficient.

Assuming that the expected return within the capital market investment portfolio is \mathcal{R}_m , the rate of return of the risk-free investment is α_i , the contribution rate of the i investment to the market portfolio risk is β_i , and the change of the investment i itself is \mathcal{E}_i , then according to the CAPM, the rate of return \mathcal{R}_i of the i real estate investment can be expressed as for:

$$\mathcal{R}_i = \alpha_i + \beta_i \mathcal{R}_m + \mathcal{E}_i \quad (3)$$

Equation (3) indicates that the rate of return on a single investment is driven by two factors: the first component is the change in the state of the whole real estate market, and the second factor is the change in the condition of the investment itself. The following is a statement of the statistical principle:

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_\delta^2 \quad (4)$$

According to this formula, the degree of uncertainty associated with the rate of return (also known as risk) is defined by two factors, market fluctuations $\beta_i^2 \sigma_m^2$ and investment fluctuations σ_δ^2 (non-systematic risk). Hence, using the non-diversifiable risk metric β_i , the entire risk of the i asset may be divided into systematic risk and unsystematic risk.

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_\delta^2 = [\sigma_i^\varepsilon]^2 + [\sigma_i^u]^2 \quad (5)$$

Among them, σ_i is the standard deviation of the return rate of the i investment; σ_i^ε is the systematic risk of the i investment ($=\beta_i \sigma_m$); σ_i^u is the unsystematic risk of the i investment [8, 9].

The systematic risk of each individual investment and the degree to which investments are correlated with one another are what ultimately define the total risk of a portfolio investment. Use $[\sigma_p^\varepsilon]^2$ to represent the systematic risk of the real estate investment portfolio, and X_i represent the weight, then

$$[\sigma_p^\varepsilon]^2 = \sum_{i=1}^n X_i^2 \beta_i^2 \sigma_m^2 + \sum_{i=1}^n \sum_{j=1}^n X_i X_j \beta_i \beta_j p_{ij} \sigma_m^2 \tag{6}$$

The expected rate of return of portfolio investment \mathcal{R}_p is the weighted average of the expected rate of return of each investment, that is,

$$\mathcal{R}_p = \sum_{i=1}^n X_i \mathcal{R}_i = \sum_{i=1}^n X_i a_i + \sum_{i=1}^n X_i \beta_i \mathcal{R}_m \tag{7}$$

Thus, based on Formula 7, the model II can be derived:
Thus, the following objective functions are:

$$\max \mathcal{R}_p = \sum_{i=1}^n X_i a_i + \sum_{i=1}^n X_i \beta_i \mathcal{R}_m \tag{8}$$

$$\min [\sigma_p^\varepsilon]^2 = \sum_{i=1}^n X_i^2 \beta_i^2 \sigma_m^2 + \sum_{i=1}^n \sum_{j=1}^n X_i X_j \beta_i \beta_j p_{ij} \sigma_m^2 \tag{9}$$

With the restrictions of: $\sum_{i=1}^n X_i = 1, X_k \geq b_2, X_s \geq b_3, \sum_{i=1}^n T_i \leq T_0, X_i \geq 0, i = 1, 2, \dots, n$.

5. Practical application

The impact of real estate investors' selection of portfolio investments on reducing absolute risk is demonstrated using a straightforward scenario. Example: A developer finances real estate ventures and categories A and B of real estate investments are included in its investment portfolio. The table displays each investment's value, projected return rate, and variance [10].

Table 1. Expected returns, variance and values.

Investment projects	Expected returns	Variance	β
A	14%	0.9%	1.2
B	10%	0.5%	0.8

A developer invests in real estate projects and has two different types of real estate investments in its investment portfolio: types A and B. Table 1 displays the expected return rate, variance, and value of each investment. The variance of the markets expected return rate $\sigma_m^2 = 0.4\%$, and the correlation coefficient between the two investments $p_{12} = 0.2$. According to the derived formula $\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_\delta^2$, the total risk of two investments can be decomposed into the sum of systematic risk and unsystematic risk respectively.

Table 2. Total Risk Breakdown.

Investment projects	Systematic risk	Non-systematic risk	Total risk
A	0.576%	0.324%	64%
B	0.256%	0.244%	51.2%

As shown in Table 2. The calculation findings indicate that the two investments, A and B's respective total risks contribute 64% and 51.2%, respectively, to the projected return and risk of the investment portfolio, while the remaining risks are scattered throughout the investment portfolio. To

make calculations easier, the weights assigned to investment A and B are taken as X_1 and X_2 , respectively. This assumes that the investment portfolio has no other constraints besides the total investment limit. Model II is substituted using these values, and the resulting mathematical model for making portfolio investment decisions is obtained by sorting and transforming the data.

$$\max f_1(X) = 0.14X_1 + 0.1X_2 \quad (10)$$

$$\min f_2(X) = 5.76X_1^2 + 2.56X_2^2 + 1.536X_1X_2 \quad (11)$$

The risk coefficient α can be included in the model by introducing α . Assume $\alpha = f_1(X)/f_2(X)$, the economic importance of this is that a real estate investment portfolio is more profitable when the risk per unit of expected return is lower. Therefore, the model mentioned above can be simplified as such.

$$\min \alpha = (5.76X_1^2 + 2.56X_2^2 + 1.536X_1X_2) / (0.14X_1 + 0.1X_2) \quad (12)$$

Consequently, this approach's economic significance is finding the best possible solution for the investment portfolio that minimizes risk while obtaining a unit of expected return. Solving this problem allows real estate enterprises to determine the optimal ratio of investments that will achieve this goal. In this case, the optimal ratio is determined to be $X_1 = 0.3$ and $X_2 = 0.7$. With this portfolio, the rate of return is 11.2%, and the return variance is 0.2095%, which is significantly lower than the risk of each investment. This demonstrates the effectiveness of a real estate investment portfolio in diversifying and reducing risks. Real estate enterprises can achieve better risk-adjusted returns and reduce their exposure to individual asset risk by investing in a portfolio with the right mix of assets. This can lead to improved profitability and stability in their investment portfolio.

6. Conclusion

Using modern portfolio theory, investors can systematically identify and manage risks, optimize their asset allocation, and achieve higher returns than investing in individual assets alone. While real estate markets can be complex and subject to various challenges, modern portfolio theory provides a valuable framework for navigating these challenges and constructing portfolios optimized for risk and return. Considering MIT when facing practical situations in the proposed research, its performance suggests strong effectiveness in diversifying and reducing the risks. However, the shortcomings of modern portfolio theory in real estate investing are severe. As real estate data is sometimes unstandardized and unconfirmed, obtaining it may be challenging. This can make it challenging to calculate an individual property's risk and return, particularly in developing or opaque markets. It might be difficult to create diverse portfolios optimal for risk and return due to the need for more data. Investors could therefore be forced to rely on estimations of asset risk and return, which might not be a fair reflection of the underlying values of the properties. However, real estate ownership necessitates a high level of managerial experience and understanding, including leasing, maintenance, and property management. This can limit the ability of individual investors to construct and manage their portfolios and increase their reliance on professional managers or investment vehicles. The complexity of real estate investment can also make it difficult to accurately estimate future returns, as changes in management or market conditions can significantly impact property performance.

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