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Full Length Article

A novel approach to using modern portfolio theory

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Abstract

Since their inception, modern portfolio theory (MPT) and the Sharpe ratio have been among the most popular investment methodologies. Although MPT has shortcomings, it effectively uses market sentiment to predict low-risk, high-earning portfolios. Our study reviews the current practice of using the Sharpe ratio or its derivative, the Sortino ratio, and suggests that investors could earn higher returns using Sterling and Treynor ratios, instead. We find that these two ratios offer higher-performing portfolios, and their statistical distributions have indicators that assist investors in determining when to use them. These new methods outperform current indexes and funds and are more robust than the capital asset pricing model used to evaluate investment performance. We conclude by suggesting additional research with different Sterling and Treynor ratios and advanced optimization algorithms.

JEL classification: C15; G11; G15; G41

Keywords: Modern portfolio theory; Portfolio screening; Risk-reward ratios

1. Introduction

This article discusses our novel use of modern portfolio theory (MPT) with different risk-reward ratios. Currently, investors use MPT with risk-reward ratios such as the Sharpe or Sortino ratios, and, at the end of an investment period, they evaluate their portfolio performance using ratios such as the Sterling and Treynor ratios. We pose the question: "Why would investors use some ratios to propose a portfolio and test the performance with different ratios at the end of the period?" To answer this question, we look at the Sharpe, Sortino, Sterling, and Treynor ratios and calculate portfolios with each ratio over quarterly intervals for five years. We used the Sharpe and Sortino ratios as industry standards in MPT and derived ways to use the Sterling and Treynor ratios similarly to these standards. Having found positive results, we propose using MPT with ratios such as the Sterling and Treynor ratios and describe our implementation of the models.

Investors have many ways in which to select portfolios, including predicting prices based on the news, investor sentiment, and company financial reports. Many other computational optimization models exist, such as the capital asset pricing model (CAPM), price/earnings ratio models, and multicriteria decision models. Investors can employ a combination of methods depending on their subjectivity. The advantages and disadvantages of these models are well documented (see Basilio, de Freitas, Kämpffe, & Rego, 2018). In this study, we use MPT because it is a popular, well-understood model used in the industry. Moreover, because our goal is to propose using end-of-term ratios such as Sterling and Treynor, MPT offers a robust platform for testing performance against the mainstay Sharpe and Sortino ratios. This is the background environment for explaining a new view of portfolio optimization.

MPT and risk-reward ratios assist investors in evaluating all assets in the same way. This, in turn, allows investors to evaluate many assets together and find portfolios that minimize risk while increasing expected returns. Ideally, investors should

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understand as many elements of risk as possible. The elements of risk are embedded in the many data sources available to investors (news, sentiment, financial reports). Therefore, using one method, such as interpreting the news, might lead to overlooking essential risk information from other sources. Hence, investors who base their portfolios only on the news may not have the same views as investors who base their portfolios on financial reports.

Moreover, several investors might look at the same data and have different views based on their choice of sources, investment horizons, strategies, or personal subjectivity. Hence, having a method that considers all investors' views (market sentiment) is advantageous. MPT (using price volatility as risk) is indifferent to the number, weights, or types of risk variables used in other models. Furthermore, because it considers price volatility, MPT inherently incorporates the net effect of market sentiment. In this way, MPT has the broadest view of what investors think about the market.

However, MPT is not without fault. Although Sharpe (1994) considers this approach a complete view of an asset's performance, it uses historical data in proposing a portfolio, which could be problematic (Rice, 2017, pp. 13–18; Schulmerich, Leporcher, & Eu, 2015). MPT theory and Sharpe assume that investors' views and goals are homogeneous. However, MPT might not account for recent market activity. Further, investors may not behave in the same way, resulting in different portfolios. Hence, the suggestions by MPT could be incorrect.

Additionally, because MPT uses historical data, it does not forecast future events. Some events become known, and market sentiment corrects for them, but others remain unknown until they severely affect the market. Investors must be aware that this could happen and affect their portfolios. Further, when used with the Sharpe ratio, MPT is criticized for its assumption that volatility is normally distributed (see Hawley & Lukomnik, 2018; Schulmerich et al., 2015). These additional assumptions have been tested in many studies in which the effects of different statistical distributions such as skew or kurtotic distributions are evaluated (see Elton & Gruber, 1997; Schulmerich et al., 2015).

Other studies simulate distributions, finding that negative skewness and leptokurtic (positive kurtosis) distributions are favorable (Xiong & Idzorek, 2011). Investors may find superior gains with negatively skewed distributions, which lead to returns above the average, whereas positive kurtotic distributions offer lower risk. Xiong and Idzorek (2011) find that tests with different distributions outperform MPT with standard assumptions in some cases. Further studies test various hypotheses, showing that investors have different return horizons (Hawley & Lukomnik, 2018). They find that homogeneous expectations can be disregarded, but MPT still yields efficiency. We review our results under these criteria to establish the performance of our models.

This article is part of a more extensive study conducted to improve the performance of sharia-compliant investments in South Africa. As such, we compare our performance measure against the South African Sharia Index. Further, to accommodate sharia requirements, we constrain the MPT model, adding only sharia-compliant assets and limiting optimization to long-only portfolios.

We further test the robustness of our models against an industry-standard CAPM. We found that we could improve portfolio performance by applying different ratios' predictions depending on market conditions. We also found that ratios like the Treynor ratio predict different portfolios from the other ratios. Treynor is beta focused, which could explain the phenomenon. This observation is explored here and offers further research opportunities.

The remaining sections develop the background models, theoretical basis and interpretation of the risk-reward ratios used to create our novel approach. The article concludes with our findings and further research suggestions. Section 2 explores the current industry MPT use describing its strengths and weaknesses. Section 3 develops the model building in our interpretation. Section 4 offers details of the risk-reward ratios calculations and how we used the data. Section 5 discusses our findings, focusing on unique scenarios and offers future research opportunities. We conclude our article in section 6.

2. Background: modern portfolio theory

Modern portfolio theory (MPT) and the Sharpe ratio are investment tools widely used in industry (Contreras, Lizama, & Stein, 2016, pp. 1–25). Harry Markowitz (who created MPT) and William F. Sharpe (the Sharpe ratio) are Nobel prize winners. Although the Literature does not indicate why their theories are the mainstay of industry, the fact that they won Nobel prizes might suggest why these models are used together. Investors could use subjectivity and sentiment when picking methods and theories, and the attention from the Nobel prizes won by these economists might influence investor choice. We argue that MPT is a comprehensive analysis tool and is suitable as an introduction to our novel approach. After the fundamentals of the approach are understood, other optimization tools can be applied.

MPT uses utility and risk to determine the most efficient investment portfolios by changing the asset weights (ω_i), to create a portfolio whose total weight is one. The portfolio equation is $E(R_p) = \sum_{i=1}^{n} \omega_i E(R_i)$, where $E(R_p) = 1$. Applying MPT with different risk-return ratios results in different efficient frontiers for the same assets. MPT uses covariance between assets when calculating an efficient frontier and the associated optimal portfolio (Contreras et al., 2016, pp. 1–25; Markowitz, 1952).

The ratios interpret risk and reward differently, influencing how an investor's portfolio might grow (their alphas) with respect to the market (Goetzmann, Ingersoll, Spiegel, & Welch, 2002; Rollinger & Hoffman, 2013). This is evident in the shape of the MPT efficient frontiers produced by the ratios. When the degree of difference in the shape of a portfolio's efficient frontiers from that of the market is greater, the portfolio shows greater independence.

The shape of efficient frontiers represents the risk for all possible portfolios. The slope between the market proxy (hurdle rate) and the efficient frontier identifies the optimal portfolio. The slope indicates risk fluctuation sensitivity, in which a steeper line indicates more volatility in returns within a risk band. Further, if the market proxy changes, the slope also changes. Hence, the risk sensitivity and optimal portfolio change, with the corresponding alphas and betas of the expected returns (Uppal & Zaffaroni, 2016). Ideally, the market's efficient frontier is beta. Any change in the shape of the portfolio's efficient frontier could be attributed to alpha. Any portfolio not on an efficiency frontier implies that for a set of assets, either more returns could be earned for the same risk or the risk itself could be lower, meaning the portfolios could be inefficient.

Different ratios for the same set of assets provide different efficient frontiers and portfolios.[1] The slope of the tangent lines shows how the returns' sensitivity varies for changes in risk. Because each ratio's risk (denominator of the ratio) is interpreted differently, a direct comparison may not make sense. Because it is possible to calculate all four ratios for each portfolio, we can find a common comparison platform and compare the results of all the models.

However, MPT has some inefficiencies and faults. Although Sharpe (1994) considers this approach a complete view of an asset's performance, it uses historical data in proposing a portfolio, which could be problematic (Rice, 2017, pp. 13–18; Schulmerich et al., 2015). MPT theory and Sharpe assume that investors agree that future returns are based on historical performance and that investor outcomes are all the same (homogeneous). However, MPT might not account for recent activity (e.g., breaking news).

Further, when used with the Sharpe ratio, MPT is criticized due to its assumption that the volatility is normally distributed and that market sentiment can be assumed (e.g., Hawley & Lukomnik, 2018; Schulmerich et al., 2015). Many studies have tested different statistical distributions such as skew or kurtotic distributions (e.g., Elton & Gruber, 1997; Schulmerich et al., 2015), showing changes in expected outcomes. Further studies have simulated distributions, finding that negatively skewed and leptokurtic distributions are beneficial (Xiong & Idzorek, 2011). This makes sense because investors can achieve higher gains with negatively skewed distributions since the distribution favors above-average returns. Positive kurtotic distributions suggest that expected returns variance is lower than normally distributed returns. Xiong and Idzorek (2011) find that the tests with different distributions outperform MPT with standard assumptions. However, their studies use unconstrained models. Hence, we test their findings in our constrained models.

Moreover, the Sharpe ratio's strengths and weaknesses are thoroughly discussed in Section 4. Although they indicate varied results in the portfolios proposed, many studies do not significantly raise the results over an extended period. This was not the case using our novel approach, suggesting that using derivations of our ratios can offer new insights into portfolio dynamics and better performance.

3. Theoretical literature review

Because MPT is a theoretical model, it makes several assumptions. Some assumptions hold in our constrained model discussed here, while others might not.[2] MPT is a model designed to provide perpetual growth while minimizing risk (Markowitz, 1952).

The MPT model is defined as:minimize $\sum_{i=1}^{n}\sum_{j=1}^{n}\omega_i\omega_j\ \sigma_{ij}$ (risk), and maximize the expected return,

$$\mathrm{E}(R_p)=\sum_{j=1}^{n}\omega_i E(R_p),$$

where:

R_x is the expected return (simplified notation for $E(R_p)$ as noted above)

R_p is the return on the portfolio p,

R_i is the return on asset i

ω_i is the weight ω of component asset i

σ_{ij} is the covariance between assets i and j (Markowitz, 1952).

The model assumes that all investors have homogeneous expectations and that this will not change over time. Risk is expressed statistically as volatility, and efficiency is calculated with the shared interaction (covariance, σ_{ij}) between assets. However, some theoretical assumptions exist when interpreting market sentiment as volatility. The assumptions imply that investors' outcomes are homogeneous, and distributions have probabilistic returns with the lowest normally distributed volatility (Hawley & Lukomnik, 2018; Rice, 2017, pp. 13–18; Schulmerich et al., 2015). Ideally, the assumptions are that all investors calculate a risk-return portfolio for a single period (for all time).

Further, it assumes that volatility is normally distributed, and we can make statistical inferences assuming that the distributions are normal. However, historical data show that the distributions might not be normal and that market sentiment changes over time (Hawley & Lukomnik, 2017). Additionally, some investors create portfolios such as hedge funds with highly skewed or kurtotic data (Davies, Kat, & Lu, 2016). Hence, market sentiment could influence the appearance of the distributions of the assets. Studies have shown that the effects of correcting for normal distributions do not always yield significantly better results (Elton & Gruber, 1997; Schulmerich et al., 2015). Given that, investors who rely on MPT might subjectively choose to review the shape of the distributions

[1] Investors could also obtain different efficient frontiers using the same ratio in an unconstrained or differently constrained MPT model.

[2] Ordinarily, in the purest form, investors input data for all investment assets into MPT in order to determine the portfolio. MPT assumes perpetual growth and includes short selling. However, we base our constraints on Sharia requirements, using three main criteria. First, we limit assets to those that fit Islamic ideology; second, we rebalance the portfolio if an asset became noncompliant with Islam; and, third, because Islam prohibits short selling, we constrain the model to long-only portfolios.

when picking portfolios. We discuss the shape of the distributions in Section 5.2 and find that they can enhance prediction.

In considering data, some studies show that high-frequency risk data can increase performance (Ma, Li, Liu, & Zhang, 2018). We chose to use daily data. There are approximately eighty daily and sixteen weekly data points when rebalancing quarterly. Setting the assumption of a normal distribution aside for a moment, daily data maximize the size of the data set and provide more accurate distributions. In that way, more data gives us a more precise view with which to analyze the normality assumption when determining the skewness and kurtosis of the distribution. However, investors do not have homogeneous expectations and might not follow suit.

Further, assets can be traded at different frequencies, making the optimal analysis frequency dynamic. Hence, the analysis done with this frequency could differ from analysis done with other frequencies. Therefore, after our approach is understood, research in more comprehensive optimization algorithms can be used to test different frequencies in the assets and the MPT analysis.

Moreover, our study is based on sharia assumptions. Therefore, our interpretation of the MPT differs from other studies in three ways. First, we do not simulate short selling in the market. Second, we limit assets to those that are sharia compliant, and, third, we use the South African Sharia Index (SASI) instead of the standard market proxy used by other investors. Fourth, unlike other studies, we deduct the annual compulsory religious alms called zakat when reporting results. The Literature discusses using the risk-free interest rate as a common starting point when calculating an optimal portfolio. This homogeneous expectation does not directly apply to Islam since Islam outright rejects interest. Instead, we use the consumer price index (CPI) with *zakat* (2.5%) as our hurdle rate (CPI + 2.5%). We also use a rate that, at a minimum, equals inflation after subtracting *zakat*.

Preliminarily, because we looked at sharia investment in South Africa in a broader study, we compare our results to those with the South African Sharia Index (SASI). Fig. 1 shows that, although it slightly underperforms, the SASI is comparable to the South African FSTE All-Share index. The graph plots the performance of South Africa's major indexes and our best-performing portfolio. The SASI (yellow line) performs similarly to the Top 40 and All-Share indexes, albeit generally lower, as shown from June 2015 to September 2019. Our best-performing portfolio (the green line) consistently outperforms all the indexes throughout the period. The statistical and economic significance in the results is discussed in Sections 5.2 and 5.3, respectively, in the findings chapter.

In sum, although MPT remains one of the tools at the disposal of investors, it is governed by assumptions and has material inefficiencies and human bias. Some studies claim that sentiment models (in behavioral economics) are more adept at incorporating human bias (e.g., Simo-Kengne, Ababio, Mba, & Koumba, 2018). However, we argue that they are implied in the net effect of volatility. Like most models, MPT fails to use the latest market information effectively or to make speculative assumptions that could cause unpredictable behavior. Noting these limitations, we show that using MPT can offer high-performance portfolios. To use MPT as an investment tool, investors must accept the principles and assumptions on which MPT is built. That is, all price changes are reflected in the volatility (Markowitz, 1952). It is up to the investor to consider what affects an asset's price volatility. If investors can accept these assumptions, they should be able to rationalize the use of risk-reward ratios and MPT. Further, MPT allows investors to relax constraints that may depart from homogeneity and could affect performance.

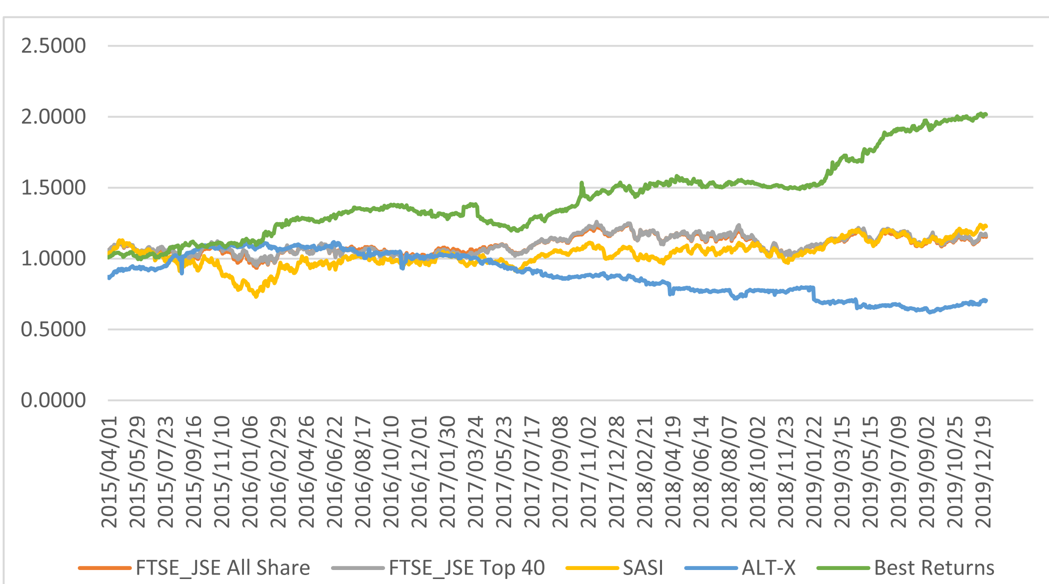

Fig. 1. Comparison of the performance by South African indexes and the best portfolio.

4. MPT ratios and variables

Originally, MPT was devised as an economic theory to optimize the possible risk of any additional utility earned. Moreover, MPT with risk-return ratios is the most popular quantitative investment method used. The risk-return ratio terminology is commonly understood. Typically, returns are calculated as the returns of an asset minus the risk-free returns ($R_x - R_f$). However, we use $ln(p_t/p_{t-1})$ to give a relative view of change.

4.1. Sharpe and Sortino Ratio

The most commonly used ratios are constructed as follows:

$$\text{Sharpe Ratio} = \frac{ln(p_t/p_{t-1})}{\sigma_x},$$

where:

$ln(p_t/p_{t-1})$ is the expected return

σ_x is the standard deviation of the expected return (Sharpe, 1994)

The Sortino ratio is different from the Sharpe ratio in that it only uses downside standard deviation.

$$\text{Sortino Ratio} = \frac{ln(p_t/p_{t-1})}{drawdown(\sigma_x)},$$

where:

$ln(p_t/p_{t-1})$ is the expected return

$drawdown(\sigma_x)$ is the standard deviation of negative returns (Rollinger & Hoffman, 2013)

The Sharpe and Sortino ratios are well known and written about widely. Their strengths and weaknesses are the subject of research from their introduction to date (e.g., Goetzmann et al., 2002; Kaplanski, Levy, Veld, & Veld-Merkoulova, 2016; Rollinger & Hoffman, 2013; Scholz, 2007; Sharpe, 1994).

To implement the ratios and MPT simulations, we use Microsoft Excel (Excel). The numerator in this study is the average daily returns earnings net of the hurdle rate.[3] Average daily returns are calculated as $average\left(ln\left(\frac{closing\ price_t}{closing\ price_{t-1}}\right)\right)$, and t is the date. The denominator is $\sqrt{\frac{\sum(cp_t - average(cp))^2}{count(cp)}}$, for all t in the Sharpe ratio, and cp_t is capped at zero for the Sortino ratio, so only negative volatility is considered. The advantages and disadvantages of Excel are discussed in Section 6, which discusses the algorithms.

The Sterling ratio is ordinarily used at the end of a period and not used in MPT. In our study, we reinterpreted the ratio to use it as a periodic risk-reward ratio. There are several interpretations of the Sterling ratio (e.g., Kolbadi & Ahmadinia, 2011; Magdon-Ismail & Atiya, 2004). This ratio is like the Sortino ratio, which uses drawdown data for the denominator. However, the numerator or expected return is interpreted differently. The Sterling ratio used in this study is adapted from:

$$Sterling\ ratio = \frac{CARR}{|ALD|},$$

where:

$CARR$ is the compound annualized rate of return.

ALD is the average largest drawdown (Magdon-Ismail & Atiya, 2004)

The ratio proposes using compound annual rates of return ($CARR$), instead of $ln(p_t/p_{t-1})$. The risk is measured as the average largest drawdown (ALD). In some cases, 10 percent is subtracted from the ALD to account for bond yield rates. Sterling proposed this when the ratio was defined (Kolbadi & Ahmadinia, 2011), but it is optional since we use a hurdle rate. Further, the original period was defined as one year. However, the periods used in the numerator and denominator are flexible as long as both are calculated within a fixed period of analysis (Kolbadi & Ahmadinia, 2011; Magdon-Ismail & Atiya, 2004).

No research detailing the use of the Sterling ratio in MPT could be found. If we calculated $CARR$ using the entire period, we could not calculate covariance. Therefore, we adapted the ratio in two ways to work in MPT. First, because we rebalance the portfolios, we calculate $CARR$ over monthly periods, such that, for a one-year period of analysis, there are twelve $CARR$ values. Second, when calculating the ratio, we use daily ALD. We use daily drawdowns because we include daily data in our analysis and the previous two ratios. This frequency aligns with the finding that high-frequency risk data supports better portfolio performance, and testing different frequency data is recommended to determine efficiency (Ma et al., 2018). With these changes to the Sterling ratio, we could calculate covariance.

Hence, the difference between the Sterling ratio and the Sortino ratio is in the numerator in which the analysis uses $CARR$. Interpreting the ratio in this way maintains the purpose of the Sterling ratio. $CARR$ smooths out the returns and expresses the expected returns as a compound investment return.

$$CARR = frequency\left(\left(\frac{end\ price}{start\ price}\right)^{1/(t*frequency)} - 1\right),$$

where t is the number of years.

The Treynor ratio was defined to evaluate a portfolio's performance against market risk (Hübner, 2005). Ideally, the market has a specific Treynor ratio over a period, and the Treynor ratio measures a portfolio's susceptibility to changes in the market. As with the other ratios, if an investor's portfolio has a higher ratio, it could outperform the market for the same risk or conversely earn equivalent returns while reducing the effect of market risk (market beta risk). Beta is calculated as $\beta_a = \frac{cov(R_a, R_m)}{var(R_m)}$, where R_a is the returns on assets, and R_m is the market returns. When the Treynor ratio is calculated, each

[3] Daily is chosen because it is convenient for creating large data sets. It is easier to determine distribution normality with larger data sets. Daily could have been any subjective period like a weekly period.

asset's beta is calculated, and the weighted portfolio has the combined beta risk (β_x) of all the assets. Each asset's beta may be unique and contribute uniquely to the portfolio's beta, depending on their weight. Beta is calculated using both upside and drawdown volatility, so the interpretation suffers from the same issues as the Sharpe ratio, in which upside volatility may increase total volatility. Therefore, further studies should look at a drawdown equivalent to beta. Similarly, ratios that use alpha should also be considered in further studies.

For this study, the Treynor ratio uses the standard beta interpretation. The ratio is:

$$\text{Treynor Ratio} = \frac{ln(p_t/p_{t-1})}{\beta_x},$$

where:

R_x is the expected return

R_f is the risk-free rate (market proxy)

β_x is the portfolio's beta of the expected return (Hübner, 2005)

The Treynor ratio also needs to be interpreted for use in MPT because betas are generally calculated over an entire period. We calculate betas at monthly intervals over an investment period to derive the covariance of betas. Calculating betas for a weighted portfolio is a common practice (Pettengill, Sundaram, & Mathur, 1995). However, no evidence could be found to calculate (up to 12 in our case) betas within a period. In using the Treynor ratio, our study optimizes beta-risk adjusted returns, in which our Treynor ratio is $\frac{ln(p_t/p_{t-1})}{covariance\ (\beta_x)}$.

The critical difference in the Treynor ratio is that it uses beta (systemic risk). By minimizing only beta, the MPT model allows any level of alpha risk. Generally, performance yields vary in different market scenarios (Ashraf, 2013; Ashraf & Mohammad, 2014; Masih, Kamil, & Bacha, 2018). Without knowing the alpha, investors may take unacceptable risks that they would not have taken after reviewing the total risk. Alternatively, if assets are highly correlated to beta, Treynor's results will be similar to those of the index (South Africa Sharia Index in our case), yielding similar results. Therefore, we calculate all 4 ratios for all 19 portfolio iterations and compare the results to understand the findings further.

Each ratio is subject to the same criticism, namely, that it implies that previous performance predicts future performance. Investors may find that recent activity is not reflected accurately in forecasting performance. However, although these criticisms are valid, the ratios have particular merit in quantifying portfolio performance and considering market sentiment to give investors insight into how other investors may view assets. The differences between the ratios are in how returns and risk are interpreted. The Sharpe, Sortino, and Treynor ratios consider average growth over a period, whereas the Sterling ratio considers a compound return. Both can be used to calculate returns over future periods. Further, each ratio has a unique interpretation of risk. These differences are illustrated in Table 1, which shows the variables used for calculating the four ratios. The numerator is derived as a form of returns for each ratio, and the denominator is derived as some form of associated risk.

Table 1
List of variables derived and where they are used.

	Sharpe	Sortino	Sterling	Treynor
Returns (Numerator)				
Expected returns	X	X		X
Compound annualized rate of return			X	
Risk (Denominator)				
Standard deviation	X			
Drawdown standard deviation		X	X	
Beta of the expected return				X

4.2. Data collection, cleaning, and derivation

The data source for the ratios is daily price data collected from the Johannesburg Stock Exchange (JSE Marketing, 2013) or Yahoo Finance (Yahoo, 2021). All assets were collated in a single Excel spreadsheet. Missing data were imputed as an average using the day before and after. If there were a few days of missing data, each missing data point was calculated incrementally (or decrementally) from the day before. We excluded the asset if any data was missing more than 5 data points. The imputation calculation used is:

$$cp_{t+1} = cp_t + \frac{cp_n - cp_1}{n-1}, \text{for all missing data}, m,$$

where $t = 1.\ m\ and\ n = (m + 2)$

All the data were converted to South African cents (currency), and the returns were normalized. The initial total portfolio was made 100 cents. Thereafter, because MPT relies on complete historical data during each period of analysis, only assets for which all the data were available for that period were used. Further, because we perform long-only constrained MPT analysis, the data were screened for assets with positive returns ($(p\ end\ date) - (p\ start\ date) > 0$) to manage the size of the data set.

5. Analysis

We use three months and stock exchange news to confirm whether assets remain sharia compliant and to rebalance the portfolio. Hence, we rebalance the portfolios every three months over five years (starting January 2, 2015). Investors can choose to rebalance at different intervals. The first three iterations have less data; after that, we use 252 trading days. In some cases, less than 252 trading days remain in the calendar year. To compensate, we use data from the previous December.

Table 2
Number of data points per period.

Iteration	Period length	Analysis Period	Number of data points
1	Three months	January to March 2015	63
2	Six months	January to June 2015	123
3	Nine months	January to September 2015	187
4	One year	January to December 2015	252

Thereafter, we used 252 of the latest data points.

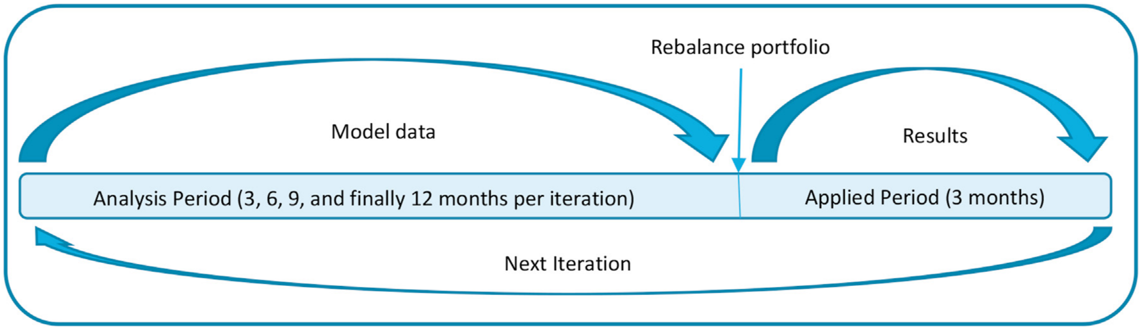

Fig. 2. Depiction of how data were used to model portfolios.

Table 2 shows the periods used and the number of data points per period. We had 63 data points for the first iteration, 123 for the second, 187 for the third and 252 data points thereafter.

The simulations are repeated every three months, and the portfolios are amended with the new suggested assets. The results are used to derive returns for the next three months. The results of the three months going forward are taken as actual returns that the portfolio would have yielded. The analysis and rebalance timeline are illustrated in Fig. 2.

The preliminary tests include comparing the various MPT models and current South African indexes and products. A difference of two means *t*-tests is used to evaluate the models' expected returns and volatility (DeFusco, McLeavey, Anson, Pinto, & Runkle, 2015). Because we use historical data in the study, we can evaluate our results with actual data in the following period, answering the performance evaluation questions proposed.

Each ratio predicts different weighted portfolios. Because the volatility interpretation for each ratio is different, a straight comparison of the model's portfolios is unfounded. Therefore, all the ratios are calculated for all iterations, allowing us to analyze the risk and results for all the models. We obtain efficient frontier curves, tangent slopes, and loss probability analysis.

After we complete our simulations, we discuss our findings and compare them to the established indexes and funds in the next section. Additionally, we analyze the distributions for the next period to reveal predictive trends. We then compare our robustness results to those from the CAPM. The CAPM is calculated using various coefficients, and the same betas calculated for the Treynor ratio. We benchmark the errors of our MPT models and the CAPM with the same data to discuss the statistical and economic significance. Our findings are discussed in the next section.

6. Findings

Simulating performance with historical returns does not guarantee future returns. However, they offer a way to understand the behavior of assets and portfolios to assist investors in decision-making (screening). After each simulation, we select the results that seem to fit our highest returns and lowest risk drawdown criteria. We find that we did not always choose the best options. Table 3 presents the summary statistics for the analysis. The last two rows tabulate the results of our subjectively chosen portfolio and the best possible portfolio we could have found based on our simulations. The best returns portfolio assumes that an investor could always select the best-performing portfolio after each iteration.

The "Final Returns" column shows real growth from July 1, 2015, to December 31, 2019. Except for Treynor, our MPT simulations outperform the current funds and indexes. The

Table 3
Summarized comparison of results at the end of the period of analysis.

	Final returns	Net return after *zakat* and CPI	Std Dev (Risk)	**Std Dev DD (Drawdown risk)**	Sharpe ratio for the index or portfolio	Sortino ratio for the index or portfolio
Current indexes and funds:						
SASI	118.68%	−10.38%	1.14%	0.75%	−9.14	−13.82
AltX	80.96%	−48.10%	1.12%	0.84%	−42.83	−57.36
ABSA	93.59%	−35.47%	2.63%	1.84%	−13.48	−19.24
Kagiso	128.94%	−0.12%	0.65%	0.42%	−0.19	−0.28
Our MPT simulations:						
Sharpe	146.40%	17.33%	0.88%	0.59%	19.61	29.49
Sortino	149.24%	20.17%	0.94%	0.61%	21.37	32.81
Sterling	159.26%	30.20%	1.01%	0.65%	29.86	46.55
Treynor	114.99%	−14.07%	1.06%	0.69%	−13.22	−20.24
Subjective	133.60%	4.53%	0.91%	0.60%	4.99	7.52
Best Returns	201.66%	72.60%	0.74%	0.47%	97.82	153.14

Note: Red indicates negative returns against a five-year inflation-adjusted increase. The inflation-adjusted value of 100 cents is R1,2906 at the end of the analysis period.

Table 4
Summarized comparison of results.

	Subjective Returns	Best Returns
Sharpe	6	4
Sortino	10	1
Sterling	3	6
Treynor	0	8
Total iterations	19	19

"Net Returns after *zakat* and CPI" column shows that none of the existing funds and indexes keep up with CPI, whereas, in general, our models do. Additionally, we correct for religious alms (*zakat*) over the 5 years; hence, our comparative returns are higher.

The "Risk" and "Drawdown Risk" columns show that our risk profiles are generally lower than the current alternatives, except for Kagiso. However, MPT typically has higher returns for that risk. The last two columns show that the industry uses Sharpe and Sortino ratios. Even though Kagiso may be a viable fund, it does not add value to the investor because its returns are lower than our hurdle rate. The "Best Returns" row with a 101.66 return shows that investors could have earned 3.5 times the returns of Kagiso for similar risk. Hence, we find that our models offer better performance, and we continue to discuss how to select portfolios to improve returns. Net return after *zakat* and CPI are a magnitude larger since the Kagiso fund almost breaks even while the best portfolio returns 72.6 percent.

Table 4 presents our choices for obtaining a Subjective or Best Returns portfolio as tabulated in the last two rows of Table 3. For Subjective Returns, we constantly chose ratios with lower drawdown volatility, and when the drawdown was similar, we picked higher returns. Hence, in the nineteen iterations, we chose the Sharpe ratio six times, Sortino ten times, and Sterling three times. However, if we had had better foresight, we could have selected ratios that yield the Best Returns portfolio. As in Table 4, the Best Returns portfolio primarily has Sterling and Treynor predictions, meaning other screening tools are needed. The Literature suggests that statistical distributions can infer portfolio suggestions. Therefore, we perform an in-depth analysis of the results and their statistical information to understand the results of the ratios. We find that investors could trade off slightly higher risk and select the Sterling ratio. However, it was not so easy with the Treynor predictions.

6.1. Understanding Treynor results

The Treynor ratio yields higher returns in 8 of the 19 iterations. Unlike Treynor's predictions, Sharpe ratios are consistently lower than the others, and the drawdown risk is consistently higher; hence, it is never picked. Moreover, the statistics show that the Treynor model is the most volatile.

Fig. 3 shows that Treynor (gray line) predicts better returns with more data. In 2015, with less data, it predicted the worst-performing portfolios twice. In 2016 Treynor was the "Best Returns" portfolio choice. However, in 2019, when all the other ratios predict high-performing portfolios, Treynor predicts lower-performing portfolios. Treynor asset weights are different from those of the other ratios. This contributes to Treynor's unique performance and higher volatility. For example, in some periods Treynor predicts the worst-performing portfolio, and the other ratios also perform poorly. However, the portfolio predicted by Treynor recovers entirely in the next period, but the other ratios do not. Table 5 shows the sample data and returns over these periods.

Although the values are not important, the difference in portfolio weight choices and the results are. It appears that Treynor can predict a somewhat unique portfolio in which a

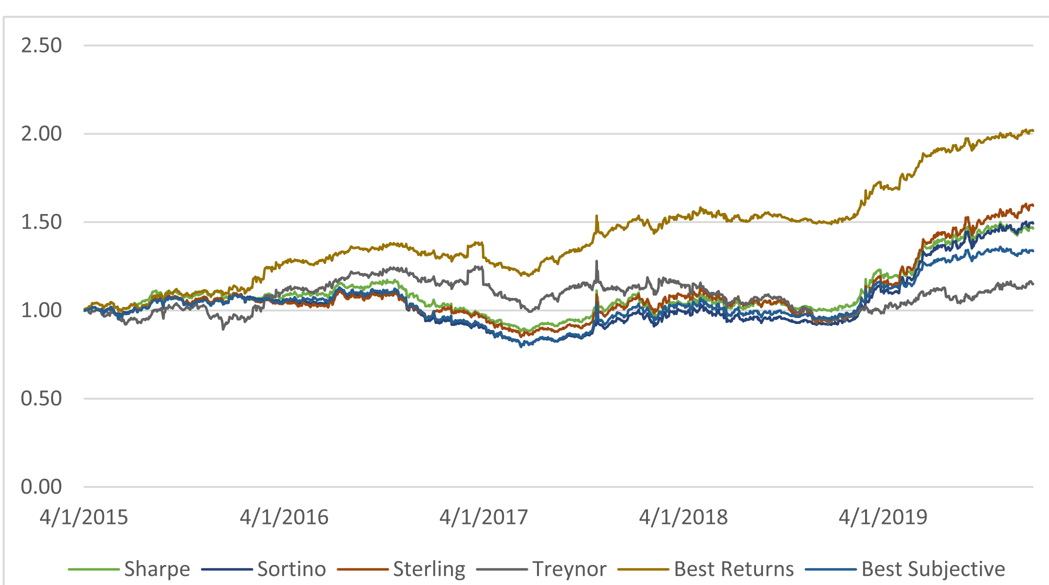

Fig. 3. MPT results over the full period.

Table 5
Example of unique Treynor predicted portfolios.

	April–June 2017				July–Sept. 2017			
	Asset 1 wt	Asset 2 wt	Asset 3 wt	returns	Asset 1 wt	Asset 2 wt	Asset 3 wt	returns
Sharpe	21.7%	9.8%	7.5%	−7.6%	14.0%	10.8%	10.8%	5.4%
Sortino	21.1%	7.9%	6.7%	−8.5%	13.2%	11.1%	10.3%	3.8%
Sterling	22.9%	8.7%	7.1%	−8.5%	14.5%	11.0%	10.9%	4.0%
Treynor	9.8%	5.4%	4.5%	−12.1%	6.7%	6.7%	6.7%	14.9%

loss could quickly be recouped. To show the principle, Table 5 presents the weights of the three top assets selected by each model over the April–June 2017 period, when a significant loss occurred. The market recovered in the next period (July–September 2017). The Treynor portfolio recovers the most in the next period. Longer-term studies could investigate whether Treynor predictions are good for recovery scenarios.

Treynor is different from the other three ratios in that it optimizes betas. Betas can give insight into why Treynor outperforms 8 out of 19 times. Betas are calculated for all four ratios. The data are grouped into betas, in which Treynor should have been chosen (8 times) and when they should not have been. Correlation and R-squared (coefficient of determination) statistics are calculated to test whether the two groups differ. Table 6 tabulates the results.

Although the data sample may be small, the table shows that Treynor produces the best portfolio when the other three ratios have a relatively strong negative correlation. This is observed in the data, in which Treynor recovers from a previous loss, but the other ratios do not. There is a strong positive correlation when Treynor is not the best solution. Similarly, R-squared shows that when Treynor is the preferred model, a small portion of the model's performance can be attributed to the benchmark SASI (where SASI represents the beta). The case for the 11 other iterations is different. The correlation and R-squared suggest that when the Treynor model predicts a portfolio that is relatively divergent from the other three, it could yield higher returns and vice versa when they are correlated. This could be investigated further.

6.2. Predictiveness and distributions

Thus far, we have discussed the best performance using the four ratios. However, we did not look at their consistency in terms of their predictions. MPT assumes portfolios with known risk-reward ratios in perpetuity. We rebalance every three months. Hence, we do not know whether the suggested portfolio would have stabilized to the prediction over time. Instead, we evaluate the similarity of the predictions and actual results in our 19 interactions. Using Excel's paired sample of means *t*-test, we accept the null hypothesis for all four ratios, indicating that the predictions and actual results could be significantly different.[4] The findings suggest that, theoretically, the MPT perpetuity assumptions may not apply when portfolios are rebalanced over shorter periods.

Table 6
Summary statistics and unique predicted Treynor portfolios.

		Sharpe	Sortino	Sterling	Treynor
Treynor as the best solution (8 iterations)	Correlation	−0.44	−0.49	−0.53	1
	R-squared	0.20	0.24	0.28	1
Other ratios as the best solution (11 iterations)	Correlation	0.98	0.78	0.76	1
	R-squared	0.96	0.61	0.57	1

Further, we conduct two sets of paired one-tailed z-tests. The first set is between the SASI and the six MPT models, and the second set is between the Kagiso Fund and the six MPT models.[5] The null hypothesis is that the MPT models are the same as the SASI and Kagiso Fund (control samples). Because all the *p*-values are less than our alpha, we reject all the null hypotheses, meaning that our models outperform the control samples. Another interesting observation is that the Best Returns comparisons yields a zero p-value. It could be attributed to computational rounding in Excel. However, it indicates that the results are highly significant. It appears including more Sterling and Treynor iterations in the best-performing model has the most significant results, warranting further study of the Sterling and Treynor ratios.

In line with the Literature, we calculate the kurtosis and skewness of the 19 iteration results. The comparison is between the ratios in question (six Sterling or eight Treynor) and any of the other three ratios as a group—as in the Literature, the Sterling results, where leptokurtic, mostly have negatively skewed distributions.[6]

Table 7 shows the summary statistics for Sterling's kurtosis and skewness. The average and standard deviation of the six times Sterling is chosen as the best choice are compared to the 13 times when the other ratios are chosen. Sterling is a good choice if the average distribution is negatively skewed, but the standard deviation's skewness might not be suggestive. The Treynor results differed from the Literature in that the platykurtic distribution predicted portfolios with higher returns.

Table 8 presents the summary statistics showing that the average kurtosis and skewness are negative, meaning that the distribution has fatter tails with more data on the positive side of the average. Similarly, the kurtosis and skewness of the standard deviations are markedly lower when Treynor is the

[4] Where the *p*-values were less than our alpha (0.05).

[5] The six MPT models include the four ratios' portfolios and the Subjective and the Best Results portfolios. The Kagiso fund is the best performing sharia-compliant fund. Other funds could be used, depending on investor subjectivity.

[6] As in Table 4, when choosing the best-performing portfolio, Sterling would have been chosen six times.

Table 7
Sterling ratio distribution: summary of calculations.

	Average		Standard deviation	
	Sterling	Other ratios	Sterling	Other ratios
Kurtosis	0.0031	−0.5857	0.0033	0.4805
Skewness	−0.7040	0.0839	0.4371	0.3879

best choice. The negative skewness predicts a higher probability of higher-than-average returns in both cases. The current Literature suggests that only leptokurtic distributions are predictive, but we found evidence that platykurtic distributions could also predict favorable portfolios. Further studies with bigger sample sizes could provide more precise suggestions. The platykurtic, negatively skewed distribution should also be investigated for other ratios. In practice, investors who want to use this screening method could analyze the current data and apply it one period ahead, as depicted in Fig. 2.

6.3. Robustness testing

We used the modified CAPM to test for robustness. The model is reproduced here with the error (ε_{it}) as the subject of the equation:

$$\varepsilon_{it} = -\alpha_i - \beta_{i0}(Rm_t - Rf_t) - \beta_i^{'} Z_{t-1}(Rm_t - Rf_t) + R_{it} - Rf_t$$

where:

R_{it} is excess return on asset i in the period t (net of the risk-free rate)

α_i is the abnormal performance of asset i

$(R_{it} - Rf_t)$ is excess returns of the portfolio over a benchmark at time t

Rm_t are excess returns on the benchmark asset

$(Rm_t - Rf_t)$ is the excess return of the benchmark at time t

β_{i0} is the systemic risk for the asset i for the current period

$(\beta_i^{'} Z_{t-1})$: $\beta_i^{'}$ is the vector of conditional beta related to the time-lagged predetermined instrument Z_{t-1}.

ε_{it} is the forecast error for asset i at time t.

The model uses historical data for each of the variables. $\beta_i^{'}$ is time-lagged. Hence, we exclude the first iteration. For consistency, the historic betas used are the same covariate, weighted betas derived in the MPT models. By making ε_{it} the subject of the equation (see above), we effectively take the current performance and subtract all other calculated performance factors. The error is then the unexplained performance. We create a similar measure for MPT. Hence, we calculate the errors as the difference between actual and predicted returns. The errors could be negative if the actual returns are lower than the predicted or time-lagged return estimations. To calculate the magnitude of errors, we take the absolute value of the error as a percentage of the actual return.

Table 8
Treynor ratio distribution: summary of calculations.

	Average		Standard deviation	
	Treynor	Other ratios	Treynor	Other ratios
Kurtosis	−0.8038	0.2261	0.6975	3.8901
Skewness	−0.0431	0.1897	0.5023	0.8946

Table 9
Examples of unique Treynor predicted portfolios.

	Sharpe	Sortino	Sterling	Treynor
Average Magnitude: MPT	18.46%	20.35%	22.27%	17.18%
Average Magnitude: Modified CAPM	29.85%	28.32%	29.73%	40.73%
Standard Deviation: MPT	11.11%	13.26%	13.15%	14.53%
Standard Deviation: Modified CAPM	16.42%	15.48%	19.29%	21.96%
Correlation between errors	0.35	0.27	0.01	−0.31
r^2	0.12	0.07	0.00	0.10

Table 9 presents the results of the MPT and CAPM magnitude of errors. MPT gives more consistent predictions when the average and standard deviation of the magnitude between the 18 iterations is lower than that of the modified CAPM. Next, we test for correlation between MPT and the modified CAPM. The last two rows show that the degree of correlation and the R-squared determination between the two methods are relatively low. Treynor is negatively correlated to the CAPM. It stands to reason, as the CAPM uses betas, and Treynor minimizes beta. We can conclude that although both methods (MPT and modified CAPM) have some errors, the MPT models tend to be more accurate and more robust than the CAPM.

As established in this study so far, Treynor could yield better results if it becomes more evident when to pick a Treynor-based portfolio. First, we consider a paired sample for a means *t*-test between MPT and CAPM errors in conducting the analysis. However, because we rebalance the MPT every three months, which affects the predictions into perpetuity, we exclude the *t*-test analysis. Further research with different beta frequencies may yield more insight and should be investigated.

Economic significance is better for noting practical importance without being prescriptive (Mitton, 2021). Unlike other scientific undertakings where the order of magnitude is considered by factors of 10, we look at how growth and returns are reported. Industry practice reports growth as a percentage of the initial investment. In our case, we questioned the significance of our 3.5 times over performance of our best portfolio against the Kagiso fund. We considered our results economically significant. First, indexes and current funds do not omit *zakat* when reporting their results. Hence, these benchmarks are overreported at face value. Omitting *zakat* from the SASI decreases returns to 8.3 percent over five years, whereas our worst-performing portfolio grows by 15 percent. Our returns are 1.8 times the SASI and could subjectively be deemed economically significant. Similarly, Kagiso outperformed the worst-performing fund by 1.9 times. Furthermore, the best-performing portfolio is a magnitude of 12 over the *zakat*-adjusted SASI and 3.5 times more than the Kagiso fund.

Similarly, based on the results in Table 9, we deem the standard deviation's magnitude of errors between our results and the CAPM (between 1.4 and 2.3 times in favor of our results) economically significant. The test for robustness and economic significance of returns and errors shows the need for further research on portfolio optimization.

6.4. Future research

To the best of our knowledge, this is the first paper to use the Sterling and Treynor ratios in MPT in this way. Hence, throughout the discussion, we note avenues for further research. To sum up, further research to test different frequency data, consider less-constrained or unconstrained MPT models, or derive different versions of the ratios will enrich knowledge on this subject.

For example, Sharpe and Treynor use total volatility, whereas Sortino and Sterling use drawdown volatility. The use of drawdown beta volatility with the Treynor ratio could be analyzed. The model becomes:

$$Drawdown\ Treynor\ Ratio = \frac{R_x - R_f}{Drawdown\ \beta_x}$$

where:

R_x is the expected return

R_f is the risk-free rate (market proxy)

$Drawdown\ \beta_x$ is the drawdown betas of the expected return

Similarly, because Sterling is the next-best model, a hybrid Sterling-Treynor ratio could provide further insight into portfolio optimization. The model would be:

$$Hybrid\ Ratio = \frac{CARR}{\left|covariance\left(Drawdown\ \beta_x\right)\right|},$$

where:

$CARR$ is the compound annualized rate of return.

$covariance\left(Drawdown\ \beta_x\right)$ is the covariate matrix of drawdown betas

Further, we note the limits of Excel as an optimization tool.[7] However, Excel is often used for research and discussed in the Literature (see Google Scholar, 2022). We find Excel advantageous because the user interface and tabular layout offer researchers a good visual platform for performing the calculations and understanding how our new models differ from prior models. The study is about improving performance, and our results support improvements with this novel approach. Further research focused on optimization with specialized languages such as Python, R, or MatLab will be advantageous. These languages can easily accommodate a more complex algorithm that considers dynamic frequency periods. The algorithms would have two distinct parts.

The first part of the algorithm accommodates that different assets are traded at different frequencies. Hence it calculates different frequency risk-reward ratios for each asset to determine their most favorable frequencies. The second part focuses on calculating covariances and optimizing the best rebalancing frequencies. The algorithm is iterative and is illustrated in Fig. 4.

The various ways in which frequencies for the ratios can be calculated are shown in Appendix 2. These various ways of using the data introduce increasing levels of algorithmic complexity and statistical inference. The complexity level increases as different frequencies of all assets are calculated in the first part of the algorithm, and covariant matrices created for each frequency in the second part, where the same frequency is used for the entire data set. Different frequencies can be calculated for both the numerator and denominator when the compound annual rate of returns and the average largest drawdown (ALD) are calculated for the Sterling ratio, further increasing the complexity of the algorithm and the derived data size. At this stage, all the ratios for each asset can be compared, and the researcher can investigate optimal frequencies to use in the second part of the algorithm. Increasing the complexity might cause the optimal frequencies between the assets to differ.

Furthermore, different ratios may be optimal at different frequencies, further expanding the data set. Iterative algorithms can calculate the covariant matrices over all the data generated in the first part and apply them in the second part to determine the overall efficiency. Researchers will have to interrogate the data and determine how to deal with any missing data problems between frequencies. After the matrices are derived, MPT will yield solutions, as in this initial study.

The frequency analysis and algorithms discussion show several research disciplines for further study. Studies can improve portfolio predictions, optimization algorithms, and statistical inference. MPT is used in fields beyond financial analysis. There are opportunities for studying optimization more broadly, with ratio interpretations not considered before.

Policy implications are beyond the scope of this study. From the perspective of sharia, different countries could have other restrictions that can be introduced as constraints. Further, fund managers may have policies that mandate asset weighting or the number of assets in a portfolio. They can also be constrained in MPT algorithms.

7. Conclusion

In an attempt to improve investment possibilities for Muslim investors, we undertook to find high-return portfolios with minimal risk. Investors can pick portfolios in many ways, such as interpreting the news, reading financial statements, and relying on their intuition. We argue that risk-return ratios such as the Sharpe and Sortino ratios, take the net effect of all investors' positions on an asset. Modern portfolio theory (MPT), together with the Sharpe and Sortino ratios, is a common tool used to predict portfolios. However, we find that MPT could be used with other risk-reward ratios, such as the Sterling or Treynor ratios typically used for post-period evaluation.

Standard Sterling and Treynor ratios are not conducive for use in MPT; hence, we reinterpret them. We calculate periodic compound rates of return for the Sterling ratio and calculated beta as a covariance matrix for the Treynor ratio. Using these

[7] The Excel model used and the parameters set are outlined in Appendix 1.

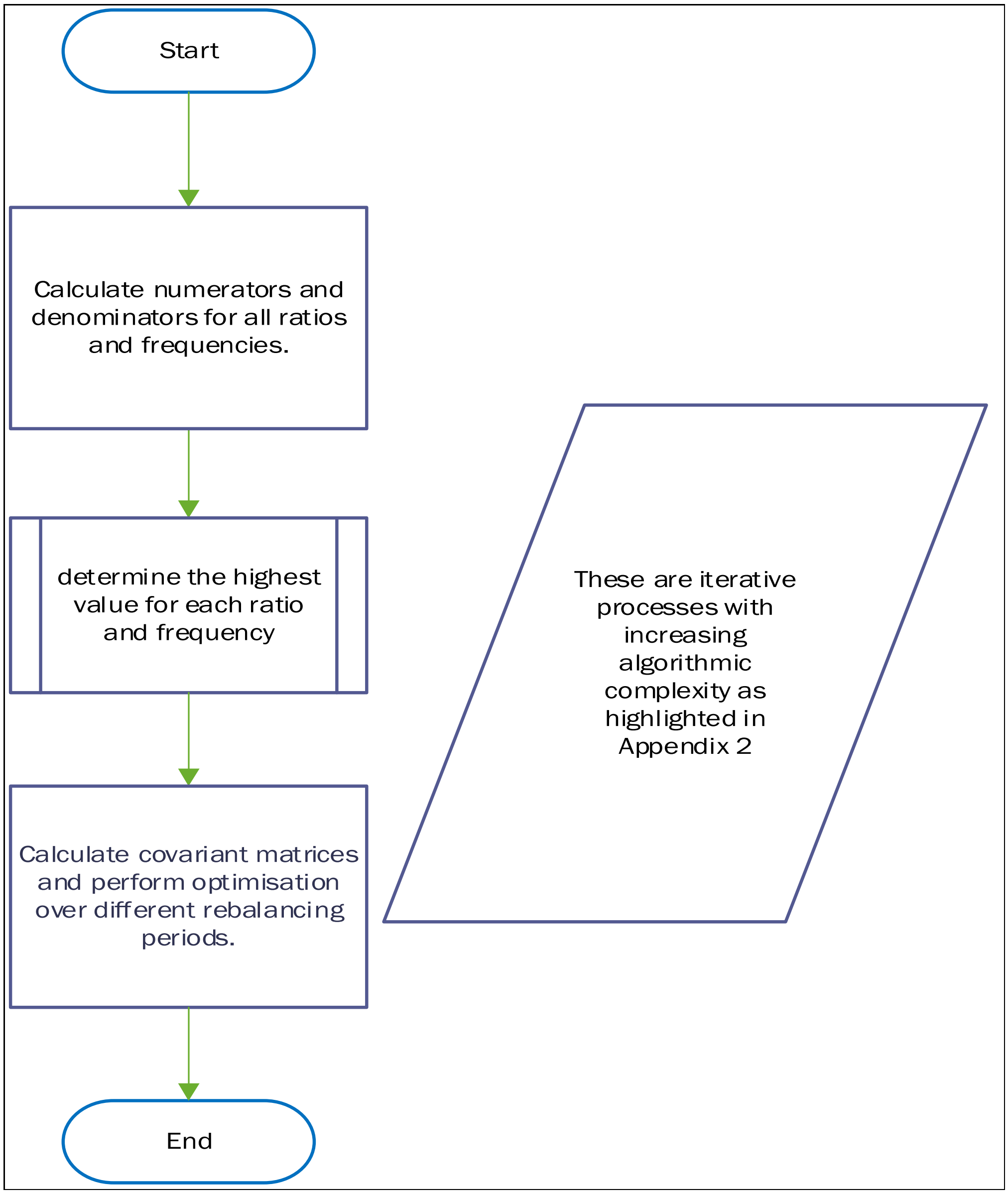

Fig. 4. MPT algorithm for optimizing different frequency risk-reward data.

calculations, we simulate five years (19 iterations) of portfolios for all four ratios, selecting portfolios to maximize returns and minimize risk.

We conclusively find that our models could outperform the current market indexes and funds and be more robust than the CAPM, with economically significant results. However, further analysis of the resulting distributions was needed to improve our selection. Using kurtosis and skewness improved predictiveness. Kurtosis and skewness play a part in selecting portfolios, and further study is warranted on how they can support economic significance. Additionally, because redefined post-period ratios proved to offer improved performance, we suggest that further studies be conducted with different versions of risk-reward ratios. Moreover, data frequency can be calculated in many ways when these ratios are applied. These ratios can also offer significant improvements and should be studied using specialized optimization algorithms. Optimization algorithms of increasing complexity can be designed in stages and tested to reveal statistical and economic significance. Lastly, we posit that, although the context of our research is the sharia environment, we believe that research in a more general environment could be conducted.

Declaration of competing interest:

The authors declare that there is no conflict of interest.

Appendix 1.

1. The MPT model in Excel.

Using the Sharpe ratio $= \frac{ln(p_t/p_{t-1})}{\sigma_x}$, for example, the optimization problem was set out in Excel using matrix multiplication (MMULT) equations. Portfolio returns are calculated as the weighted excess returns $\omega_i\, ln(p_t/p_{t-1})$ for each asset. The assets and their weights form vectors that are multiplied to yield a scalar product. The result becomes the numerator of the Sharpe ratio.

Next, the weights and covariance of returns of the assets are multiplied to derive a scalar portfolio standard deviation (the denominator). A variable is created to ensure that the weights of the assets add up to 100 percent. Similar calculations were derived for the other ratios.

The Excel optimization model uses generalized reduced gradient nonlinear optimization. We set the optimization objective to maximize the risk-return ratio by changing the weights. The optimization model weights are constrained to ≥0 (zero) to eliminate short selling (negative weights). Each asset was given equal weight as a starting point for the model. The constraint precision was set at 0.001, and the number of iterations was set at 1000. The parameters were the same for all four ratios and all 19 iterations. All the iterations converged, giving optimal portfolio suggestions.

2. Calculating ratios with different frequencies and increasing complexity.

The table shows how research can develop the optimization algorithms of this research further.

MPT model variation	Sharpe	Sortino	Derived Sterling	Derived Treynor	Other derived ratios
Single frequency optimization for both numerator and denominator for all assets	X	X	X	X	X
Optimized ratio based on a single frequency taken after calculating a range of frequencies for both numerator and denominator for all assets	X	X	X	X	X
Optimized ratio based on frequencies taken after calculating a range of different frequencies for numerators and different frequencies for denominators for all assets			X	X	X
Optimized ratio based on a single frequency taken after calculating a range of frequencies for both numerator and denominator for each asset			X	X	X
Optimized ratio based on frequencies taken after calculating a range of different frequencies for numerators and different frequencies for denominators for each asset			X	X	X

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